On Generating Combined Drop Size Distributions from Point Measurements in a Spray

K. M. Bade* and R. J. Schick
Spraying Systems Co., Spray Analysis and Research Services, Wheaton, IL, 60187, USA

Abstract

In this investigation, methods used to generate combined drop size distributions, from discrete measurement points, are developed and evaluated. A hydraulic, flat spray and an internally air-atomized, multi-orifice spray are investigated experimentally; representing common but distinctly different sprays. The details of each spray plume are captured with an extensive matrix of measurement points, which are then used to evaluate the drop size distribution difference across each spray plume, and to calculate combined drop size distributions throughout a planar cross-section of each spray. Weighted-average distributions are generated, using the local flux and discrete area at each measurement point, which are relevant physical characteristics within each spray plume. It was found previously by Bade and Schick [1] that measurements should be performed at approximately each 10% interval of the overall spray plume size in order to achieve errors less than 5% for combined spray plume statistics. Also, the measurements should be performed along both the x and y axis for a flat spray, and along a single-axis from the center-to-edge for a multi-orifice air-atomized spray for drop size investigations. The developed methods allow the logical combination of drop size distributions, collected at many points, to be used to generate holistic drop size distributions that are representative of a planar cross-section within a spray plume.

Keywords: Drop Size Distributions, Planar Spray Characteristics, Post-Processing Methods

Introduction

Significant effort has been made by many researchers to establish meaningful distributions and statistics to represent the drop size and velocity characteristics of a chaotic and complex spray plumes. For example, in the well recognized text by Lefebvre [2], and originally proposed by Mugele and Evans [3], definitions are presented for the Arithmetic Mean, Sauter Mean Diameter, and many other statistical values that can be generated from well resolved drop size distributions. These distributions may be collected at a point within a spray, or throughout a spray plume. Many methods and instruments, some of which are reviewed in texts by Tishkoff, *et al.* [4], Dodge, *et al.* [5], and Albrecht, *et al.* [6], may collect these distributions; however, a concise demonstration of the combination of multiple point-measurements into single characteristic value, or distribution, for a spray plume has not been clearly developed. Planar spray characteristics may be directly collected, by techniques such as interferometric Particle Imaging (IPI, see Albrecht, *et al.* [6]), but these methods often suffer from very limited ability to measure within a moderate to very dense spray plume. Furthermore, a significant advantage of point measurement systems, beyond the ability to characterize dense sprays, is the ability to collect *flux* information. Through this information, regions of high flux can be treated as more influential to resulting planar statistics. This is not possible with imaging techniques which only provide instantaneous or ensemble (spatial) spray measurements. The current investigation aims to use the flux information, to generate flux sensitive planar spray statistics and distributions, and builds on the previous work by Bade and Schick [7], [1].

A comprehensive review of characterization techniques developed for SAE International [8] and subsequent papers by Hung, *et al.* [9] and Gandhi, *et al.* [10] provide a list of the available measurement techniques for spray characterization and provide an impressive list of recommended tests to characterize a fuel injection spray. More recently, Fansler and Parrish [11] provide a very comprehensive review on many spray measurement instruments, the benefits and drawbacks, and describe where in the the spray they are best used. However, these investigations do not guide the post-processing of point measurements for holistic comparisons, an area which seems to be a rarely covered in published material. Bade and Schick [12] provide an investigation of PDI measurements and LSI measurements, but do not combine the results into planar values. Notably, Chapel and Hall [13] investigate a hydraulic flat spray nozzle, for the agricultural industry, and collect measurements at 225 points within a planar spray plume using a Phase Doppler Particle

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*Corresponding Author: Kyle.Bade@Spray.com

Analyzer (PDPA) instrument in order to determine the optimal measurement locations, but only perform straight-averaging and do not attempt to generate combined distributions.

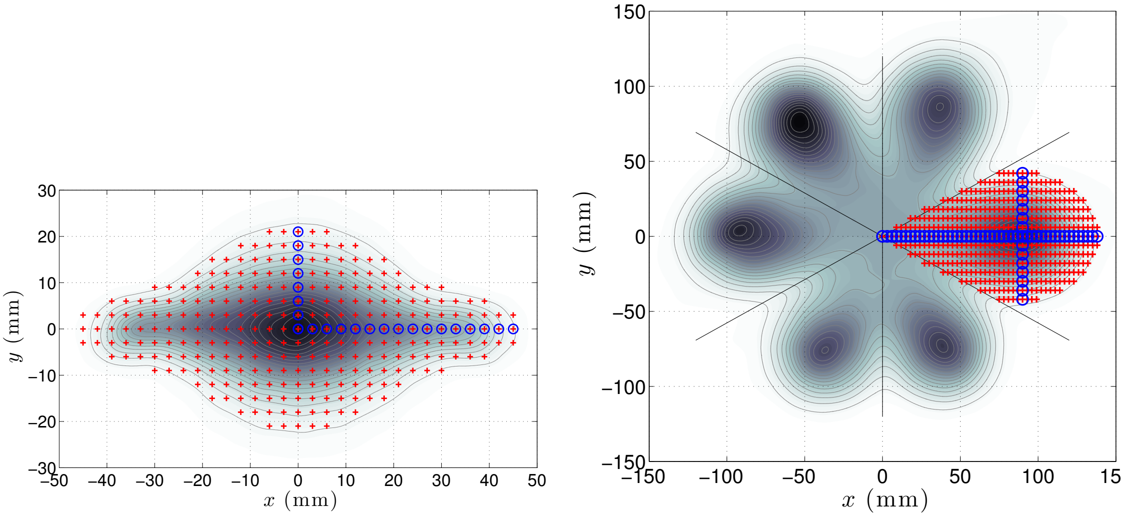
In a recent investigation, Bade and Schick [1] investigate the number of measurement points, and their locations, required to generate reliable planar statistics. It was found that acquiring measurements across both the x and y axes was necessary for a flat spray plume, but only along the major axis of a multi-orifice spray; in each case, measurements should be acquired at each 10% of the spray pattern size to achieve an error of less that 5%. The results of the present study provide methods to combine drop size distributions, collected as point measurements, into reliable holistic distributions. These methods are applied to experimental data, but could presumably be used to collapse the highly detailed data fields acquired through computational efforts for global spray comparisons.

Experimental Methods

Nozzles

Measurements were acquired for two nozzle types: *i*) a single-orifice, hydraulic, flat spray, and *ii*) a multi-orifice, air-atomized spray; both nozzles are standard Spraying Systems Co. products and are referenced herein as the *Flat*, and *Multi-Orifice* nozzle spray patterns, respectively.

The *Flat* spray was generated by a UniJet® TPU650050-TC nozzle, and provided an attractive case given that this nozzle was previously investigated in detail by Bade and Schick [14]. The nozzle was operated at a liquid pressure of 4.0 bar, resulting in a flow rate of 225 mL/min. This flat spray nozzle provides a volume distribution that is fairly symmetric about the x and y axes, but not axisymmetric. Figure 1a demonstrates this symmetry at the z=50 mm plane, and also includes an overlay of the 273 discrete measurement points acquired over the entire spray plume during PDI testing.



(a) TPU650050-TC Flat spray pattern (b) SU26B Multi-Orifice spray pattern

Figure 1: LSI result for a) the Flat spray UniJet® TPU650050-TC nozzle and b) the Multi-Orifice Air-Atomized SU26B nozzle spray, with overlaid PDI test points

The *Multi-Orifice*, internally air-atomized nozzle provided an additional, more complex, spray plume shape for further analysis of the optimal spray measurement-point location subsets for interrogation when generating planar spray statistics. Figure 1b provides the LSI result of the average spray plume at the z=178 mm plane at 2.4 bar water pressure and 2.67 bar air pressure, which resulted in water flow rate of 136 mL/min and air flow rate of 0.0756 standard mL/min. For this study, the complex shape of the spray plume generated by a single orifice was investigated. Figure 1b includes an overlay of the array of 453 discrete measurement points overtop the interrogated plume.

Laser Sheet Imaging

Laser Sheet Imaging (LSI) measurements were conducted for each nozzle in order to provide high spatial-resolution, xy planar contours representative of the spray distributions generated by each nozzle. A LaVision SprayMaster system was used to collect 200 instantaneous images for each spray. The general setup of the LSI instrument including the laser, camera, and nozzle orientation is demonstrated in Figure 2. The ensemble averaged planar contour results from the LSI testing at z =-50 and -178 mm from the nozzle exit orifices are provided for the Flat and Multi-Orifice sprays in Figures 1a and 1b, respectively; with contour lines drawn

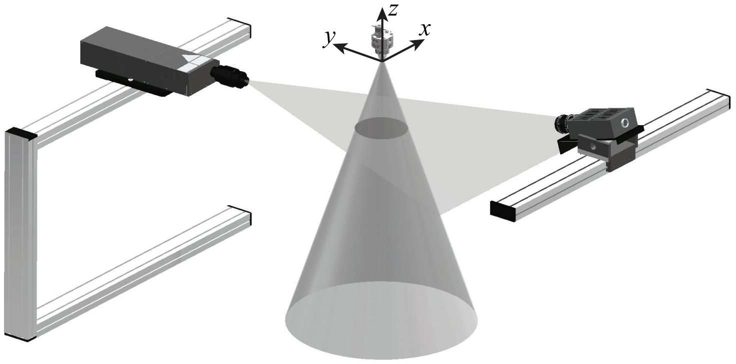


Figure 2: LSI setup demonstrating the laser (left), camera (right), and nozzle orientation

at 5% intervals of the maximum level. The relative light intensity I, in these results is scattered according to Mie theory and is representative of the spatial distribution of the surface area of all droplets passing through the laser sheet, i.e. the average Surface Area Distribution. While these distributions are not strictly representative of the spray volume distribution, these data were used to guide the selection of the locations and number of PDI test points in each spray with an attempt to acquire many measurement points within each 5% contour of each distribution. As demonstrated in Figures 1a and 1b, there are measurement points within every 5% contour line throughout each of the xy planar spray plumes. The spatial resolution of the xy plane LSI images was 0.24 and 0.56 mm/pixel for the Flat and Multi-Orifice sprays, respectively.

Phase Doppler Interferometry

Phase Doppler Interferometry (PDI) measurements were acquired, using an Artium Technologies PDI-MD-200 instrument, throughout a single xy plane of each spray plume, at the points shown in Figures 1a and 1b, to provide measurements of droplet size and velocity, as well as local volume flux. As a matter of practicality, all PDI measurement points were acquired at constant intervals from the nozzle center point. For the Flat spray, measurements were acquired at 3 mm increments in both x and y over the entire spray pattern. For the Multi-Orifice spray, measurements were acquired at every 3 mm in x and every 6 mm in y throughout one of the six plumes. Similar acquisition methods were used as those outlined as optimal by Bade and Schick [14], with the transmitter/receiver lens combination of 1000/500 mm, and the receiver located at a 40° off-axis position, which allowed for a sufficient drop size measurement range and resolution. Bade and Schick [14] also provide an extensive investigation of the accuracy of volume flux measurements using the PDI instrument which were found to be accurate to within approximately 2% when proper droplet trajectory considerations are used in the data collection methods. The ability of the PDI instrument to simultaneously measure drop size, velocity, and local flux provides an attractive source of point measurements in a spray; however, the post-processing methods outlined in this paper are applicable to spray data acquired through any appropriate point-sampling instrument or method, including computational results. The appropriateness for these methods to be used in post-processing of computational data, which inherently have access to these critical spray characteristics at a high spatial-resolution through an entire spray pattern, is very strong and an attractive implementation of these methods.

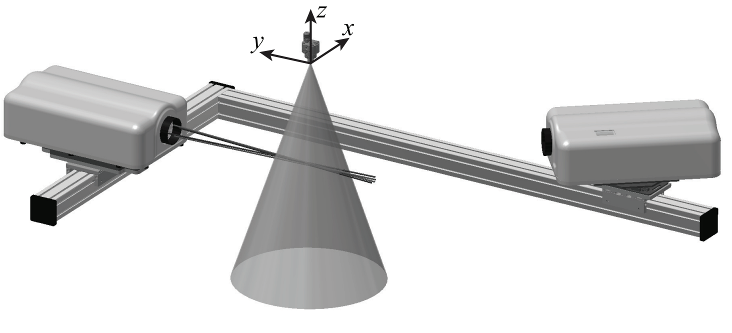
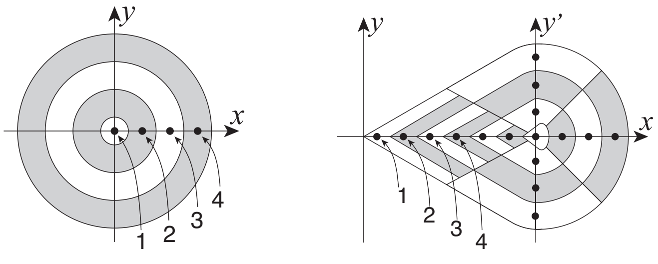


Figure 3: PDI setup demonstrating the laser transmitter, receiver, and nozzle orientation

Weighted Average Methods

Weighted averaging is a reasonable and robust method for combining many individual measurements into meaningful combined statistics; or, in this case, drop size distributions representative of a planar cross-section of a spray. The methods to generate weighted average values or distributions which are representative of the planar spray plume characteristics follow typical mathematical processes; see, for example, the work of Bade and Schick [1] or more classically in the text by Bevington and Robinson [15]. For the purposes of this paper, the necessary practical steps are presented to generate weighted averages based on the discrete measurement area that each point measurement is assumed to represent, A, and local volume flux, q_z; these values represent relevant physical characteristics of the investigated spray plumes. The use of these two parameters together provides a larger weight to the results measured where the majority of the sprayed volume is located.

Discrete area weighting accounts for the fact that each measurement point is often not expected to be representative of the same discrete area within a spray plume. An example where this will have a large influence is in the implementation for results with a round spray plume in which uniformly spaced radial measurements are acquired along a single center-to-edge line. This is demonstrated in Figure 4a, where the center-point (point #1) is expected to represent the spray characteristics of a very small circular area compared to, for example, the outer-most point (point #4) which represents a much larger area-ring. It can easily be shown that the relative area of each area-ring to the central point follows the following trend: 1, 8, 16, 24, etc. By including measurements along additional radial axes, the relative area of each discrete region is reduced, but remains non-uniform.



(a) Round or Flat spray (b) Multi-Orifice spray

Figure 4: Diagram examples of the area represented by a subset of discrete measurement locations (●) for a) a round/flat or b) the multi-orifice spray pattern shape.

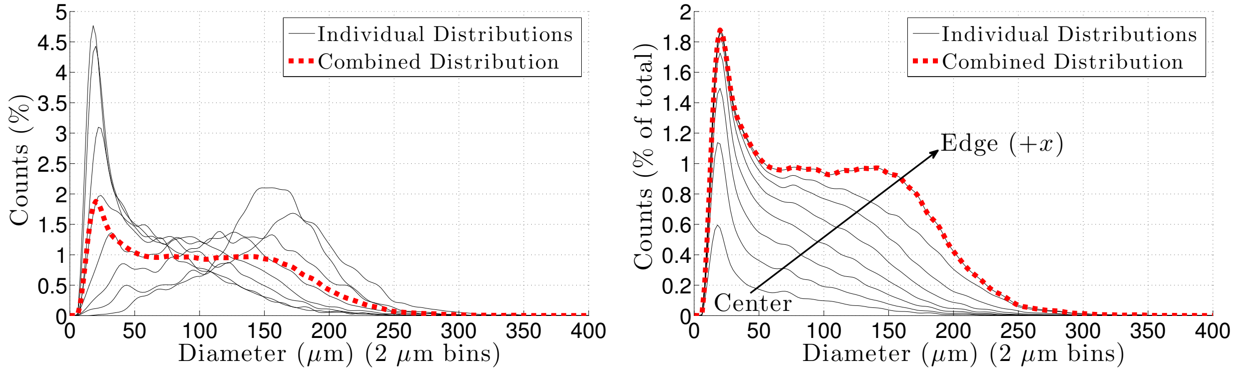
For the Multi-Orifice spray, additional considerations are appropriate to properly weight the more complex spray plume shape. Investigating the spray plume generated by one of the six orifices, the plume shape is interpreted as triangles toward the nozzle center, and semi-circles toward the outer edge of the spray, relative to the y' axis. For this non-axisymmetric spray plume shape, determining a spray *center* point can be subjective; and the nozzle center provides the only tractable starting point for regularly spaced measurements. Therefore, a method is devised to use uniformly spaced measurement points to investigate a single plume of the multi-orifice spray with measurements beginning at the *nozzle* center (x=y=0). Figure 4b provides a reasonable example of measurement points along the x and y' axes, which may be used to capture the spray statistics characteristics. In order to determine the discrete areas shown in Figure 4b, the location of maximum volume flux along the x-axis is taken as the dividing point (and location of the y' axis) for area weighting, with the nozzle center located at point #1.

While the local volume flux and discrete area represent logical weighting values, other parameters could be used or added to achieve alternate weighted results. For each parameter that is used for weighting, $P_{i,n}$, the relative weight,

$$\omega_{P_i,n} = \frac{P_{i,n}}{\Sigma_n P_{i,n}}, \tag{1}$$

is determined at each measurement point; where i is the weighting parameter index and n is the measurement point index. When incorporating multiple weighting parameters, as will be done here, a combined relative weight at each measurement location,

$$\omega_n = \frac{\Sigma_i \omega_{P_i,n}}{\Sigma_n \Sigma_i \omega_{P_i,n}}, \tag{2}$$



(a) Individual Distributions

(b) Combined Distribution

Figure 5: Drop size distributions along the x-axis of the Flat spray showing the a) the individual distributions and b) normalized distributions summed bin-wise from center-to-edge forming the combined distribution.

must be determined. This combined normalized weight at each measurement location using multiple parameters may then be used to generate a weighted average for any measured spray characteristic of interest. Note that with proper weighting across all measurement points, the sum of all normalized ω_n values will equal 1; this is also true for any $\omega_{P_i,n}$. The resulting weighted average value,

$$\overline{X_w} = \Sigma_n(X_n\omega_n), \tag{3}$$

incorporates the relative distribution of the weighting parameters along with the measured quantity at each discrete location, X_n. Bade and Schick [1] investigated the resulting statistical values, and the current investigation builds off those results to combine the drop size distributions from multiple measurements.

The total weighting parameter, ω_n, and the local drop size distributions, are used to generate planar weighted average distributions in the xy plane. Each drop size distribution, which has the measured droplet sizes organized into bins (2 μm resolution is used here) is normalized by dividing each bin-count by the total number of counts for that measurement; this is demonstrated in Figure 5a. Note that the number of bins will scale the counts (%) values as higher resolution (smaller bins) will incorporate fewer total counts in each bin.

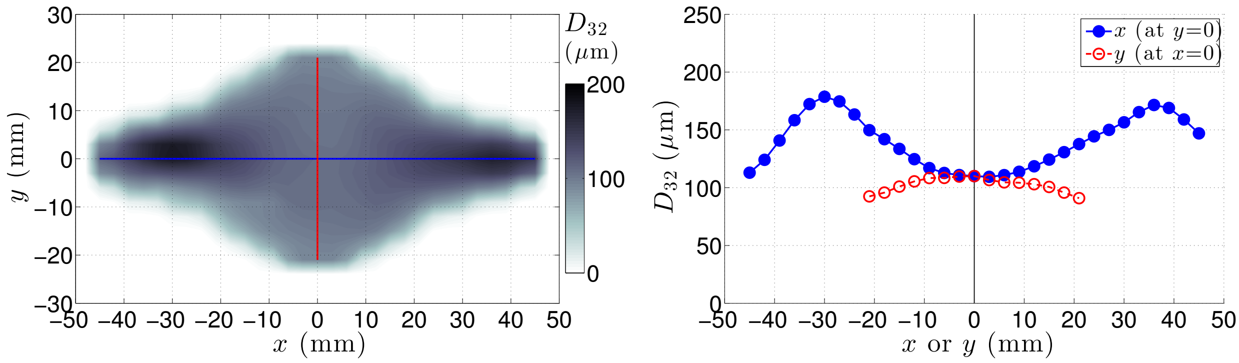
Each normalized distribution is then multiplied by the total weighting parameter calculated at each point-measurement-location. The resulting weighted distributions are then summed bin-wise to produce a combined normalized drop size distribution. In Figure 5b the weighted distributions are summed, starting at the spray centerline point, adding each subsequent distribution until the full combined distribution is presented at the spray edge; this combined distribution is also provided in Figure 5a for comparison with the individual normalized profiles. This process of combining the individual distributions could be conducted without weighting the profiles, but the result would contain an equal influence from edge points (often very little spray volume) and central point (often representing only a small discrete area).

Results and Discussion

The results of the PDI point-wise measurements for each nozzle are presented in this section; for brevity of this paper, these results focus on the Sauter Mean Diameter, D_{32}; however, the evaluations are representative of other spray characteristics (such as v_z, D_{10}, $D_{v0.5}$, etc.).

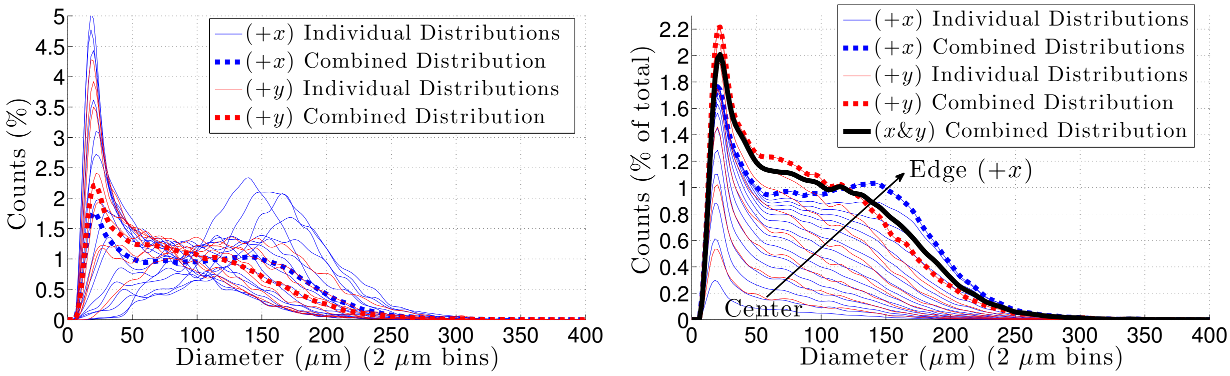
For the Flat spray, a total of 273 measurement points were collected with the PDI according to the matrix demonstrated in Figure 1a. Figure 6a provides the resulting xy-planar results, at z=-50 mm from the nozzle, for the TPU650050 nozzle spray. Figure 6b provides the extracted x and y axis profile results, which will be used for drop size distribution analysis, but also demonstrates the strong symmetry about both the x and y axes while not being axisymmetric.

The results for D_{32} in Figure 6 demonstrate that the drop size quantities are not axisymmetric, but are symmetric about both the x and y axes. The results of Bade and Schick [1] reflect this, and recommend that measurements be acquired across both the x and y axes when characterizing a hydraulic flat spray. Figure 6b demonstrates teh D_{32} profiles along both the x and y axes, and Figure 7a demonstrates the individual drop size distributions at these locaitons (note that ont eh + axis distributions are shown for clarity). Normalized, weighted, and combined drop size distributions across both the $+x$ and $+y$ axes are provided in Figure 7b, and the differences in the center-to-edge point measurements can be seen to deviate for the x-axis and y-axis results. As indicated by the data of Figure 6 the drop size distributions include a great quantity of larger



(a) (b)

Figure 6: PDI point measurement results for the TPU650050 single-orifice, hydraulic, flat spray for D_{32} a) over the xy plane and b) along the x and y axes.



(a) Individual Distributions (b) Combined Distributions

Figure 7: Drop size distributions at the 45 (31 in x, and 15 in y) individual PDI measurement points along the x and y axes of the TPU650050 Flat spray nozzle, a) normalized and b) weighted and summed to achieve a combined distribution

droplets near the spray edge along the x-axis.

For the multi-orifice spray, a total of 453 measurement points were collected with the PDI according to the layout demonstrated in Figure 1b, covering one of the six spray plumes generated by the nozzle. Figure 8 provides the xy planar results, at z=-178 mm, from these tests with the SU26B nozzle spray. Figure 8b provides the extracted x and y axis profile results, which will be used in the distribution analysis, and demonstrates the spray pattern symmetry about he y axis, but not the x axis.

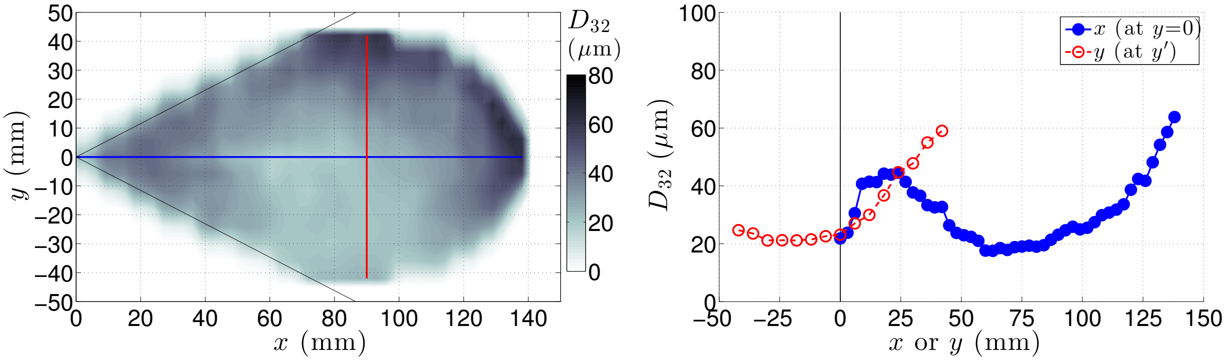
The drop size distributions for all x and y' axis measurements are provided in Figure 9. Figure 9a provides the individual normalized distributions, and demonstrates the impressively uniform drop size distributions throughout the internally air-atomized multi-orifice SU26B spray pattern. Weighting methods were shown to be unnecessary to create combined planar statistics for this nozzle [1], and this is clearly due to the uniformity of the spray characteristics across the spray pattern. Figure 9b provides the combined distributions, which demonstrate a nearly identical result using the x, y, or x & y data.

Recall, that Bade and Schick [1] demonstrate that for the Multi-Orifice spray, measurements were only required along the x-axis for accurate combined statistics, and the nearly identical combined weighted distributions using the x-only, y-only, and x & y data clearly agree with this. The combined weighted distributions recover the same drop size distribution regardless of the axis of measurement due the uniform drop size distribution generated throughout the Multi-Orifice spray pattern.

The use of weighted averaging methods to combine localized drop size distribution measurements allows holistic drop size distributions to be generated for better representation and comparison.

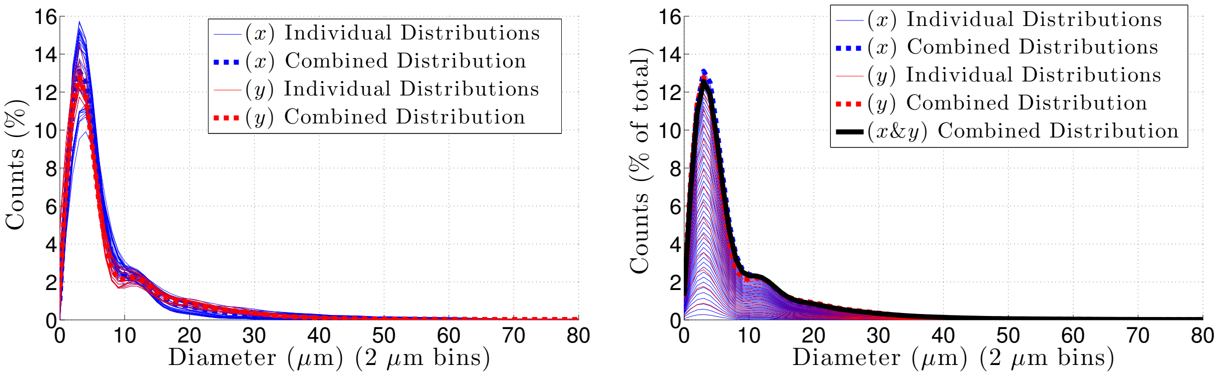
Summary and Conclusion

This study presents methods to generate combined drop size distributions from sets of point measurements using discrete area and volume flux weighting. Using the results of Bade and Schick [1] which examines weighting methods to combine point measurements into planar-representative statistics, the distributions from small on-axis measurements are shown to capture the shape of the overall distributions. These



(a) (b)

Figure 8: PDI point measurement results for the SU26B multi-orifice, air-atomizing spray for D_{32} a) over the xy plane and b) along the x-axis and y'-axis (x=90 mm).



(a) Individual Distributions (b) Combined Distributions

Figure 9: Drop size distributions at the 61 (47 in x, and 15 in y) individual PDI measurement points along the x and y axes of the SU26B Multi-Orifice spray nozzle, a) normalized and b) weighted and summed to achieve a combined distribution

methods are demonstrated to provide accurate combined distributions when the drop size distribution shape varies throughout the spray pattern (the Flat spray results), but weighting is not necessary with the spray characteristics are mostly uniform over the spray pattern (Internally air-atomized Multi-Orifice results). However, the normalization and combination methods are still useful and recommended. The finding on this investigation may serve as a guide to future researchers, to provide holistic spray drop size distributions that are accurate and representative of the entire spray plume.

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